# MIXING OF HIGH-TEMPERATURE JETS
# OF NONUNIFORM GASES

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We examine the problem of mixing of two plane-parallel (coflowing or opposed) high-temperature jets of nonuniform compressible gases. Particular solutions are obtained. In [1] the boundary-layer differential equations were used as the basis for examining the problem of mixing of two plane (coflowing or opposed) jets of nonuniform compressible gases for the condition of a linear relationship between the viscosity and enthalpy ($\mu = h$, $k = 1$), whose validity raises no question in the region of comparatively low temperatures. When considering jets heated to high temperatures, this comparatively simple relation gives way to the power law $\mu = h^k$, in which k approaches 0.5 with increase of the temperature. With use of the latter relation the resulting system of differential equations, even for uniform gases, becomes so complex [2] that for a concrete problem it can only be integrated numerically. The use of the relation proposed by Chapman and Rubesin [3] for studying high-temperature jets makes it possible to obtain the same computational advantages as in the case $k = 1$ [1], and at the same time describes satisfactorily the connection between viscosity and enthalpy over a wide temperature range.

Transition to high temperatures makes it possible to use the equation of state, since the dissociation and ionization processes which take place in gases at high temperatures lead to significant change of the gas constant R. In this case the relation $\rho = \text{const}/T$, which follows from the equation of state, becomes invalid. The relationships between the density and enthalpy for the following gases: nitrogen, hydrogen, oxygen, air, and for water have been obtained on the basis of thermodynamic calculations (using the data of [4, 5]) and the equation of state. The resulting curves of density $\rho$ versus enthalpy h from the thermodynamic calculation (points) and from the approximation $\rho = A/h^n$ are shown in Fig. 1: a) $H_2$, b) $O_2$, c) $N_2$, d) air, and in Fig. 2 for water vapor (plasma). We see that all the data are approximated with adequate precision by the expression $\rho = A/h^n$, which is a simple analytic approximation to the equation of state in which the coefficients A and n are constants and depend on the kind of gas: $H_2 - \lg A = 4$; $n = 0.77$; for $O_2$ respectively 6.1 and 0.98; for $N_2$ 5.3 and 0.862; and for air 4.6 and 0.810. Analogous relations for air and water vapor were obtained in [6].

In the following we present results of a theoretical study of the transport processes in the mixing of two plane laminar jets of nonuniform compressible gases for coflowing or opposed jet motion. We assume, as in [1], that mixing is accomplished as a result of molecular diffusion in the boundary layer (mixing zone) at the jet contact boundary and that the miscible gases do not interact chemically. All the physical assumptions of boundary-layer theory hold and, in addition, we assume that $P \neq 1 = \text{const}$, $S \neq 1 = \text{const}$ and, in addition, $P \neq S$. The latter assumptions are obviously justified by the fact that, as shown by calculations using the kinetic theory of gases [3], the influence of the gas mixture enthalpy on P is comparatively small and with increase of the enthalpy oscillates about an average value which can be taken as approximately 0.7.

We obtain the system of differential equations of steady motion of nonuniform compressible gases in the laminar boundary layer (mixing zone) of two coflowing or opposed high-temperature streams with account for the coupling relation of [3] and the approximation $\rho = A/h^n$ without account for thermo-diffusion, assuming the pressure constant throughout the flow field, in the following form:

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0, \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \quad \rho = \frac{A}{h^n}, \quad \mu = hC$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu}{P} \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial y}\left[\left(1 - \frac{1}{P}\right)\mu \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]$$
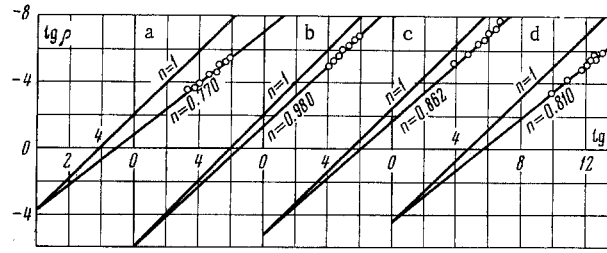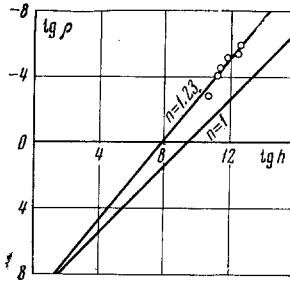
Fig. 1

Fig. 2

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu}{S}\frac{\partial C_i}{\partial y}\right)\left(C = \frac{1+h_s}{h_w + h_s}\, h_w^{0.5},\; h_s = \frac{T_s}{T\infty}\right) \tag{1}$$

($T_S = 102\,^\circ C$ is the Sutherland constant)

with the boundary conditions in the region x > 0 (x = 0 is the point where jet mixing begins)

$$u = u_1,\quad h = h_1,\quad C_i = C_1 \quad \text{for} \quad y = +\infty$$
$$u = u_2,\quad h = h_2,\quad C_i = C_2 \quad \text{for} \quad y = -\infty \tag{2}$$

After transforming (1) with the boundary conditions (2) to dimensionless form, taking as the scales the quantities associated with the undisturbed flow in the region y > 0, and then again transforming to Dorodnitsyn variables, we obtain

$$\frac{\partial u}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0,\; u\frac{\partial u}{\partial \xi} + V\frac{\partial u}{\partial \eta} = \frac{\mu_\infty}{\rho_\infty}\frac{\partial}{\partial \eta}\left[\left(\frac{h}{h_\infty}\right)^{1-n}\frac{\partial u}{\partial \eta}\right]$$

$$u\frac{\partial h}{\partial \xi} + V\frac{\partial h}{\partial \eta} = \frac{\mu_\infty}{\rho_\infty}\frac{\partial}{\partial \eta}\left[\frac{1}{P}\left(\frac{h}{h_\infty}\right)^{1-n}\frac{\partial h}{\partial \eta}\right] + (\sigma - 1)M_\infty^2\frac{\mu_\infty}{\rho_\infty}\frac{\partial}{\partial \eta}$$

$$\times\left[\left(1 - \frac{1}{P}\right)\left(\frac{h}{h_\infty}\right)^{1-n}\frac{\partial}{\partial \eta}\left(\frac{u}{2}\right)^2\right]$$

$$u\frac{\partial C_i}{\partial \xi} + V\frac{\partial C_i}{\partial \eta} = \frac{\mu_\infty}{\rho_\infty}\frac{\partial}{\partial \eta}\left[\frac{1}{S}\left(\frac{h}{h_\infty}\right)^{1-n}\frac{\partial C_i}{\partial \eta}\right]\quad V = \rho v + u\frac{\partial \eta}{\partial x},\; \sigma = \frac{c_p}{c_v}$$

with the boundary conditions

$$u = 1,\quad h = 1,\quad C_i = 1 \quad \text{for} \quad \eta = +\infty$$
$$u = u_m,\quad h = h_m,\quad C_i = C_m \quad \text{for} \quad \eta = -\infty$$
$$u_m = \frac{u_2}{u_1},\; h_m = \frac{h_2}{h_1},\; C_m = \frac{C_2}{C_1}$$

For the coflowing jet case the value of the velocity $u_2$ is positive, for the opposed jet case it is negative; $u_1$ is also always greater than 0.

Further assuming that the longitudinal velocity component, enthalpy, and concentration are functions [2] of only the variable $\zeta$ provided P and S are constant, we obtain

$$\varphi''' + \varphi''\left(\frac{H'}{H} + \frac{\varphi}{v_\infty H}\right) = 0$$

$$h'' + 2h'\left(\frac{H'}{H} + \frac{P\varphi}{v_\infty H}\right) + (\sigma - 1)\frac{M_\infty^2}{2}(P-1)\left[\frac{H'}{H}(\varphi'')^2 + (\varphi'')^2\right] = 0 \tag{3}$$

$$C_i'' + C_i'\left(\frac{H'}{H} + \frac{S\varphi}{v_\infty H}\right) = 0 \quad \left(H = \left[\frac{h}{h_0}\right]^{1-n}\right)$$

with the boundary conditions

$$\varphi' = 1,\quad h = 1,\quad C_i = 1 \quad \text{for} \quad \zeta = +\infty$$
$$\varphi' = u_m,\quad h = h_m,\quad C_i = C_m \quad \text{for} \quad \zeta = -\infty \tag{4}$$

The system of Eq. (3) obtained after the transformations is so complex that it can be integrated only numerically for any concrete problem. To obtain an analytic solution of the problem we examine the case when n = 1.
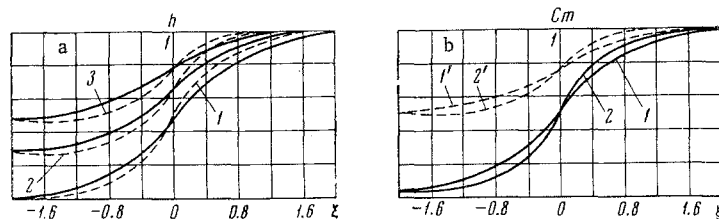
Fig. 3

Analysis of the exponent of the approximation $\rho = A/h^n$ shows that for the gases considered and shown in Figs. 1-3 the exponent does not differ markedly from 1. In any case, the considered version for high-temperature conditions is much closer to the actual situation than assumption of a linear connection between the viscosity and enthalpy $\mu = h$, where the assumption $k = 1$ in place of $k = 0.5$ is a considerably rougher approximation.

For $n = 1$ the system (3) reduces to a system of autonomous differential equations with the boundary conditions (4)

$$\varphi''' + \frac{1}{v_\infty}\varphi\varphi'' = 0$$

$$h'' + \frac{2P\,\varphi h'}{v_\infty} + (\sigma - 1)\frac{M_\infty^2}{2}(P-1)(\varphi'')^2 = 0, \quad C_i'' + \frac{S\varphi\,C_i}{v_\infty} = 0 \tag{5}$$

For the solution of the dynamic problem, as in [2], we use the iteration method of [7], writing the unknown function in series form

$$\varphi(\zeta) = \sum_{i=0}^{\infty}(m-1)^i\varphi_i(\zeta)$$

The first Eq. (5) differs from that solved in [2] only in the constant factor $1/v_\infty$, therefore we shall write the final expressions for the velocity components

$$u = \frac{1}{2}\varphi'(\zeta) = \frac{1}{2}\left[1 + \left(u_m - 1\right)\left(1 - \operatorname{erf}\frac{\zeta}{\sqrt{2v_\infty}}\right)\right] \tag{6}$$

$$V = \frac{1}{2\sqrt{\xi}}(\zeta\varphi' - \varphi) = \frac{1}{2\sqrt{\xi}}(u_m - 1)\left[\zeta\left(1 - \frac{1}{\sqrt{2v_\infty}}\right) + \frac{1}{\sqrt{2v_\infty}}\int_0^\zeta\left(\operatorname{erf}\frac{\zeta}{\sqrt{2v_\infty}}\right)d\zeta + \rho\operatorname{erf}\frac{\zeta}{\sqrt{2v_\infty}} - C_1\right] \tag{7}$$

We obtain the solution of the energy equation (6), as in [1], by the method of variation of constants, using the substitution from the first equation (5)

$$\varphi = v_\infty\frac{\varphi'''}{\varphi''}, \quad \int_{-\infty}^{\zeta}\varphi\,d\zeta = -v_\infty\int_{-\infty}^{\zeta}\frac{\varphi'''}{\varphi''}\,d\zeta = -v_\infty\ln\frac{\varphi''(\zeta)}{\varphi''(0)} \tag{8}$$

We finally obtain the general solution of the second Eq. (5) in the form

$$h = h_m + 0.5(1 - h_m)(1 + \operatorname{erf}\zeta\sqrt{2P}) + 0.25(\sigma - 1)M_\infty^2(P-1)$$
$$\times[0.5(1 + \operatorname{erf}\zeta\sqrt{2P})R(-\infty, +\infty) - R(-\infty, \zeta)]$$

$$R(-\infty, +\infty) = \int_{-\infty}^{+\infty}[\varphi''(\zeta)]^{2P}\int_{-\infty}^{\zeta}(\varphi''')^2[\varphi''(\zeta)]^{-2P}\,d\zeta\,d\zeta \tag{9}$$

$$R(-\infty, \zeta) = \int_{-\infty}^{\zeta}[\varphi''(\zeta)]^{2P}\int_{-\infty}^{\zeta}(\varphi''')^2[\varphi''(\zeta)]^{-2P}\,d\zeta\,d\zeta$$

We obtain the solution of the diffusion equation with the corresponding boundary conditions (4) with account for the substitution (8) in the form

$$C_i = \left[C_m\int_{\zeta}^{+\infty}(\varphi'')^{2S}\,d\zeta + \int_{-\infty}^{\zeta}(\varphi'')^{2S}d\zeta\right]\left[\int_{-\infty}^{+\infty}(\varphi'')^{2S}\,d\zeta\right]^{-1}$$

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or finally, with account for the expression for the function $\varphi''(\zeta)$

$$C_i = 0.5\,[(C_m + 1) - (C_m - 1)\,\mathrm{erf}\,\zeta\,\sqrt{2S}] \tag{10}$$

The nature of the gas motion, enthalpy, and concentration distributions in the mixing region of the two opposed or coflowing streams, as we see from the solutions (6)-(10), is analogous to that obtained in [1, 2], where the pattern is presented graphically and analyzed in considerable detail.

As an example we shall compare on the basis of (9) and (10) the enthalpy and concentration distribution in the mixing region of two coflowing (or opposed) streams of different compressible gases for the case of the linear relation [1] between the viscosity and enthalpy ($\mu = h$, $k = 1$) and for the case of the approximating relation $\rho = A/h^n$. Figure 3a, b shows results of the calculations of the total enthalpy h and concentration $C_i$ as a function of the generalized parameter $\zeta = \eta/2\sqrt{\xi}$ in the case P = 1 for the two versions being compared for various values of $h_m$ and $C_m$. In Fig. 3a the curves 1, 2, 3 correspond to the values $h_m = 0$, 0.3, 0.5; the solid lines are for $\mu = h$, $k = 1$, the dashed lines are for $\rho = A/h^n$. In Fig. 3b the curves 1 and 2 correspond to the values $C_m = 0$ and 0.5.

The nature of the curves shows that the effective mixing zone width when using the relation $\rho = A/h^n$ in the solution is less than when using the linear relation. We must assume that this situation is a consequence of additional turbulization of the flow, possibly as a result of dissociation and ionization at the high temperatures. Thus, the more rapid equalization of the enthalpy profile indicates more intense processes in the mixing region.

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